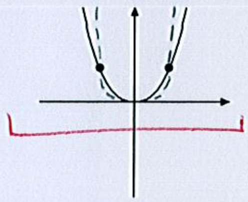
[2 PTS] Find the vertex and axis of symmetry of $f(x) = \frac{1}{4}x^2 - 2x - 12$. **ANSWER:** vertex: $(4, -16)$

axis: $x = 4$

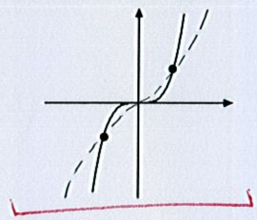
$$x = -\frac{b}{2a} = -\frac{-2}{2(\frac{1}{4})} = 4$$

$$f(4) = \tfrac{1}{4}(4)^2 - 2(4) - 12 = -16$$

[2 PTS] [a] The graph of $f(x) = x^2$ is shown below with the points $(1, 1)$ and $(-1, 1)$ highlighted. On the same grid, sketch the graph of $f(x) = x^4$.



[b] The graph of $f(x) = x^5$ is shown below with the points $(1, 1)$ and $(-1, -1)$ highlighted. On the same grid, sketch the graph of $f(x) = x^3$.



[4 PTS] $(x - 5)$ and $(x + 4)$ are factors of $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$. Using that information and synthetic division, find all real zeros of f. **ANSWER:** $5, -4, 1, 2$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline -4 & 1 & 1 & -10 & 8 & \boxed{0} \\ & & -4 & 12 & -8 & \\ \hline & 1 & -3 & 2 & \boxed{0} & \end{array}$$

OR

$$\begin{array}{r|rrrrr} -4 & 1 & -4 & -15 & 58 & -40 \\ & & -4 & 32 & -68 & 40 \\ \hline 5 & 1 & -8 & 17 & -10 & \boxed{0} \\ & & 5 & -15 & 10 & \\ \hline & 1 & -3 & 2 & \boxed{0} & \end{array}$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0 \rightarrow x = 1, 2$$

[2 PTS] Write the vertex form of the equation of the quadratic function that has vertex $(-3, 5)$ and whose graph passes through the point $(-1, 2)$. **ANSWER:** $f(x) = -\frac{3}{4}(x+3)^2 + 5$

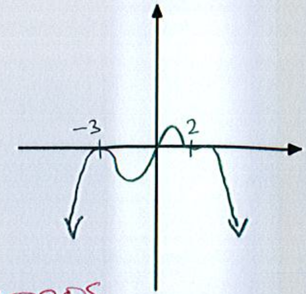
$$f(x) = a(x - {-3})^2 + 5 = a(x+3)^2 + 5$$

$$f(-1) = a(-1+3)^2 + 5 = 4a + 5 = 2$$

$$a = -\frac{3}{4}$$

ADDITIONAL QUESTIONS ON THE OTHER SIDE ➡

[3 PTS] Sketch the graph of the function $f(x) = x(x+3)^2(2-x)^5$ as shown in lecture. **ANSWER:**



ZERO	MULT	
0	1	CROSS
-3	2	BOUNCE
2	5	CROSS

LEADING TERM $-x^8$

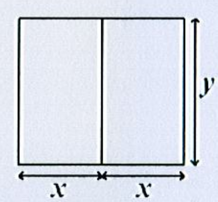
BEHAVIOR
AT $x=-3$ (1)
$x=0$ (1)
$x=2$ (1)
(-1) ANY OTHER ZEROS
(-1) WRONG LONG RUN
(-1) NOT SMOOTH + CONTINUOU

[3 PTS] Divide $f(x) = 3x + 2x^3 - 9 - 8x^2$ by $d(x) = x^2 + 1$. **ANSWER:** $2x - 8 + \frac{x-1}{x^2+1}$

Write your final answer in the form $q(x) + \frac{r(x)}{d(x)}$.

$$\begin{array}{r} 2x - 8 \\ x^2 + 1 \overline{)\, 2x^3 - 8x^2 + 3x - 9} \\ 2x^3 + 2x \\ \hline -8x^2 + x - 9 \\ -8x^2 - 8 \\ \hline x - 1 \end{array}$$

[4 PTS] A rancher has 84 feet of fencing to enclose two adjacent rectangular corrals (see figure on the right).



[a] Write the total (combined) enclosed area of the corrals as a function of x. (Your final answer must **NOT** involve y.) **ANSWER:** $56x - \frac{8}{3}x^2$

$A = 2xy$ $\qquad 4x + 3y = 84$

$y = 28 - \frac{4}{3}x$

$A = 2x(28 - \frac{4}{3}x) = 56x - \frac{8}{3}x^2$

[b] Find the dimensions of **each** corral that will produce the maximum enclosed area. **ANSWER:** $x = \frac{21}{2}$, $y = 14$

VERTEX $x = -\frac{b}{2a} = -\frac{56}{2(-\frac{8}{3})} = \overset{7}{\cancel{56}} \cdot \frac{3}{\underset{2}{\cancel{16}}} = \frac{21}{2}$

$y = 28 - \frac{\overset{2}{\cancel{4}}}{\cancel{3}}\left(\frac{\overset{7}{\cancel{21}}}{\cancel{2}}\right) = 14$